Anisotropic Properties of Strip-Type Artificial Dielectric*

N. J. KOLETTIS† AND R. E. COLLIN‡, SENIOR MEMBER, IRE

Summary—**Theoretical formulas for the propagation phase constant of a two-dimensional strip medium are presented for general directions of propagation. In addition a number of experimental results are included that verify the validity of the theory. Some of the difficulties encountered in defining equivalent dielectric constants for this medium are also pointed out.**

ARTIFICIAL dielectric media, originally introduced by Kock,[1] have received extensive investigation during the past decade. However, for the most part the detailed study of the anisotropic properties of typical structures has not been carried out. Most studies have been restricted to angles of propagation through the medium and to polarizations, so that the artificial dielectric behaved essentially as an isotropic medium. Since the anisotropic properties can be useful in practice (for example, in wave polarizers), it is worthwhile to analyze these properties for typical structures. From another point of view, anisotropic properties are usually present and hence a knowledge of these properties is needed in order to determine the actual behavior of an artificial dielectric medium in a given application.

The two-dimensional strip-type medium illustrated in Fig. 1 is a typical example of an artificial dielectric that exhibits pronounced anisotropic properties. For propagation through the medium in a direction normal to the broad side of the strips (along x axis) and with the electric field directed along the z axis, the medium behaves as an isotropic phase-delay dielectric. For the opposite polarization the medium behaves as a phase-advance dielectric. The properties of this medium with propagation restricted to be along the x-axis, has been analyzed by Brown,[2] and Cohn,[3] as well as others. If a wave with the electric field polarized in the xz plane is permitted to propagate through the medium at an effective angle θ_r' with respect to the x axis, then the medium exhibits electric anisotropic properties. The analysis for this more general case has been presented by Collin.[4] The purpose of the present paper is to give a summary of the theroetical results obtained to date, together with experimental results that show that the theoretical formulas specifying the anisotropic behavior of the medium give results of acceptable accuracy for design purposes.

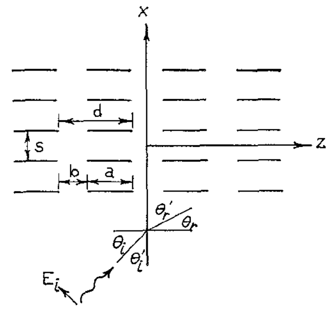

Fig. 1—Cross-section view of two-dimensional strip-type artificial dielectric, strips infinite in length along y-axis.

Modes of Propagation

Before presenting the theoretical formulas that permit the equivalent dielectric constant of the medium to be evaluated, a qualitative discussion of the possible modes of propagation in the medium will be given. If propagation is restricted to be in the x direction with the E vector along the z axis then the dominant mode is a TEM mode and all higher order modes (assumed evanescent) are TM modes with respect to the x axis. These may be labeled as $(TM)_x$ modes. If propagation is permitted to be at an angle to the x axis but in the xz plane and furthermore with E in the xz plane then all the modes are $(TM)_x$ modes. For the opposite polarization with the H vector in the xz plane all the modes are $(TE)_x$ modes. For more general directions of propagation through the medium, so that the propagation vector has a component along all three axes, the possible modes are not of the above type. One set of possible modes of propagation is still characterized by the absence of a y component of electric field, but it does have an x component of H, and hence these may be labeled as $(TE)_y$ modes, *i.e.*, transverse electric modes with respect to the y axis. These are the same types of modes that occur in connection with capacitive diaphragms in rectangular waveguides.[5] These modes may be expressed in terms of the field component H_y. Between

* Received by the PGMTT, April 13, 1961; revised manuscript received, June 30, 1961. This work was sponsored by the Electronics Res. Directorate of the AF Cambridge Res. Lab. under Contract No. AF 19(604)-3887.

† RCA Lab., Princeton, N. J. Formerly with the Case Institute of Technology, Cleveland, Ohio.

‡ Elec. Engrg. Dept., Case Institute of Technology, Cleveland, Ohio.

[1] W. E. Kock, "Metallic delay lenses," *Bell Syst. Tech. J.*, vol. 27, pp. 58–82; January, 1958.

[2] J. Brown, "The design of metallic delay dielectrics," *Proc. IEE*, vol. 97, pt. 3, pp. 45–48; January, 1960.

[3] S. B. Cohn, "Analysis of the metal strip delay structure for microwave lenses," *J. Appl. Phys.*, vol. 20, pp. 257–262; March, 1949.

[4] R. E. Collin, "Field Theory of Guided Waves," McGraw-Hill Book Co., Inc., New York, N. Y. Sec. 12.6; 1960.

[5] H. M. Altschuler and L. O. Goldstone, "On network representations of certain obstacles in waveguide regions," IRE Trans. on Microwave Theory and Techniques, vol. MTT-7, pp. 213–221; April, 1959.

any two strips the dominant mode in this class is of the form, with the z axis considered as the axis of propagation,

$$H_y = e^{-jk_0(n_y y + n_z z)},$$

where n_y, n_z are components of the unit wave normal $\boldsymbol{n}$. In addition there are field components H_z and E_x. This mode has no low-frequency cutoff.

The other class of possible modes is characterized by the absence of a y component of magnetic field and may be classed as $(\mathrm{TM})_y$ modes. These modes may be expressed in terms of E_y. The dominant mode in this class that may exist between two strips, again considering the z axis as the axis of propagation, is of the form

$$E_y = \sin\frac{\pi x}{s}\, e^{-jk_0(n_y y + n_z z)}$$

where $(k_0 n_z)^2 = k_0^2(1-n_y^2) - (\pi/s)^2$. For the usual values of s occurring in practice, $(s<\lambda_0/2)$, n_z is imaginary and this mode is below cutoff. Thus for the $(\mathrm{TM})_y$ modes the strip-medium has a low frequency stop band. Only for $s>\lambda_0/2$ can a propagating $(\mathrm{TM})_y$ mode exist, and when it does the medium functions as a phase-advance medium for these modes.

In this paper we are concerned only with the case of the $(\mathrm{TE})_y$ modes. For these, the component of the electric field along the z axis gives rise to a large z-directed electric dipole moment in each strip and hence an effective dielectric constant κ_e in the z direction. The strips are infinitely thin in the x direction; hence E_x produces no dipole polarization in the x direction, so the effective dielectric constant in this direction is unity. The dielectric constant in the y direction may be arbitrarily chosen since the $(\mathrm{TE})_y$ modes do not have an electric field component in this direction. For reasons to be explained later, it is convenient to chose the effective dielectric constant in the y direction as κ_e.

The specification of equivalent dielectric constants for an artificial dielectric medium is just a convenient artifice that will permit an analogy to be made between the artificial medium and a homogeneous anisotropic dielectric medium. Such an analogy is useful only if the propagation of a wave through the homogeneous dielectric follows the same law as that for the artificial dielectric. When this is true the refractive properties of the artificial medium are the same as for the homogeneous dielectric and the design of a microwave lens may then be based on the properties of the equivalent homogeneous dielectric. It should be noted at this time that a homogeneous medium which is fully equivalent to the artificial dielectric medium for all possible modes of propagation can usually not be specified. Thus, whatever analogy can be established is usually valid only under certain restrictive conditions on wave polarization and direction of propagation.

The homogeneous anisotropic dielectric medium chosen as an analogy for the strip-medium, for a $(\mathrm{TE})_y$ mode of propagation in the strip medium, is a uniaxial medium with dielectric constants 1, κ_e, κ_e along the x, y and z axis respectively. The basis for this analogy is that for the strip medium, the low frequency solution for the propagation phase constant β, along the wave normal $\boldsymbol{n}$, satisfies the same eigenvalue equation as for the homogeneous medium. Furthermore, for propagation in the xz plane, the extraordinary wave in the uniaxial medium has the same field components as the $(\mathrm{TE})_y$ mode in the strip medium. The analogy is, however, not a complete one, since for propagation through the medium with $\boldsymbol{n}$ having a y component, the $(\mathrm{TE})_y$ mode has a zero y component of electric field while the extraordinary wave in the homogeneous uniaxial dielectric has a zero component of magnetic field along the optic axis (x axis in the present case).[6] Nevertheless the eigenvalue equation and refractive properties of the two media are the same for the respective modes of propagation that they support.

Theoretical Results for Equivalent Dielectric Constant

The strip medium illustrated in Fig. 1 is a double periodic medium and may be analyzed in terms of modes propagating along the z axis or in terms of modes propagating along the x axis. In the first method, which will be referred to as the z-axis solution, each layer of strips presents a discontinuity at each interface. The reflection and transmission coefficients that characterize each interface are those associated with the interface between a set of semi-infinite parallel plates and free space. The latter problem has been solved by Carlson and Heins.[7] Using their results, it is found that the equivalent circuit of the strip medium for modes propagating in the z direction is that illustrated in Fig. 2.[4] The circuit in Fig. 2 is arrived at by assuming that the strip spacing s is small compared with a wavelength and that neither a nor b are so small that higher-order mode interaction between adjacent interfaces is important. For a TEM mode incident, in the xz plane, on the strip medium at an angle θ_i relative to the z axis and with the electric vector in the xz plane, the propagation phase constant βn_z through the medium is given by

$$\begin{aligned}(1-\rho^2)\cos\beta n_z d &= \cos\left[k_0(a+b\cos\theta_i) - k_0\frac{s}{\pi}(1-\cos\theta_i)\ln 4\right] \\ &\quad - \rho^2\cos\left[k_0(a-b\cos\theta_i) - k_0\frac{s}{\pi}(1+\cos\theta_i)\ln 4\right], \qquad (1)\end{aligned}$$

[6] Collin, *op. cit.*, Sec. 3.7.

[7] J. F. Carlson and A. E. Heins, "The reflection of an electromagnetic plane wave by an infinite set of plates," *Quart. Appl. Math.*, vol. 4, pp. 313–329; January, 1957, and vol. 5, pp. 82–88; April, 1947.

where

$$\rho = (1 - \cos\theta_i)/(1 + \cos\theta_i), \qquad k_0 = 2\pi/\lambda_0$$

and the other parameters are give in Fig. 1. This equation results from an analysis of the periodic structure illustrated in Fig. 2. When the frequency is low so that $k_0 d$ approaches zero, the cosine terms in (1) may be approximated according to the relation $\cos x = 1 - x^2/2$. After simplification (1) reduces to

$$(\beta n_z)^2 = k_0^2 \left[1 - \left(\frac{b}{d} + \frac{s}{\pi d} \ln 4 \right) \sin^2 \theta_i \right]. \qquad (2)$$

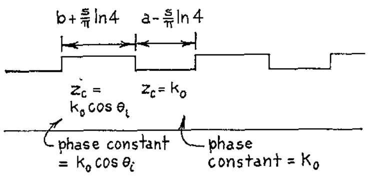

Fig. 2—Equivalent circuit for strip medium for modes propagating in the z direction.

In the present case the propagation phase constant along the x axis is $k_0 \sin\theta_i = \beta n_x$ and is known. If propagation takes place along the y axis also with a phase constant βn_y, then in (1) and (2) k_0^2 must be replaced by $k^2 = k_0^2 - \beta^2 n_y^2$ and βn_x becomes equal to $k \sin\theta_i$. The basis for this substitution is the same as that used to obtain the solution for a capacitive diaphragm in a rectangular guide from the solution for a diaphragm in an infinitely wide parallel-plate transmission line. Thus for general directions of propagation the low frequency limit of (1) is

$$(\beta n_z)^2 = k_0^2 - (\beta n_y)^2 - (\beta n_x)^2 \left(\frac{b}{d} + \frac{s}{\pi d} \ln 4 \right). \qquad (3)$$

The static solution for the dielectric constant of the strip medium in the z direction, as obtained by conformal mapping, is given by[8]

$$\kappa_e = \frac{d}{b + \frac{s}{z} \ln 4} \qquad (4)$$

when both s/a and s/b are small. Substituting this into (3) and solving for β^2 gives

$$\beta^2 = \frac{\kappa_e k_0^2}{n_x^2 + (1 - n_x^2)\kappa_e}. \qquad (5)$$

This, however, is the eigenvalue equation for the extraordinary wave in a homogeneous uniaxial anisotropic dielectric with dielectric constants 1, κ_e, κ_e, along the x, y, z axis, respectively. It is for this reason that the homogeneous uniaxial medium is chosen as an analogy for the strip medium, and also the reason why the strip medium is assumed to have a dielectric constant κ_e in the y direction. Another similar analogy is also possible, and this is discussed in the Appendix.

For high frequencies βn_z must be found from (1). By analogy with (3) the equivalent dielectric constant κ_e may then be defined by

$$(\beta n_z)^2 = k_0^2 - (\beta n_y)^2 - \kappa_e^{-1} (\beta n_x)^2,$$

or

$$\kappa_e = \frac{(\beta n_x)^2}{k_0^2 - (\beta n_z)^2 - (\beta n_y)^2}. \qquad (6)$$

In general, κ_e is a function of frequency except in the low frequency range where (4) is valid.

The solution provided by (1) may be expected to give good accuracy for small values of s. For larger values of s higher mode interaction becomes significant and should be accounted for. On the other hand, by solving the same problem in terms of modes propagating in the x direction, the condition that higher order mode interaction in this approach be negligible, is that the spacing s be large. Thus the x-axis solution is complementary to the z-axis solution in that it will yield good results for large s while the latter yields good accuracy for small s. A combination of the two methods will then enable accurate estimates of κ_e to be obtained for all values of s likely to be encountered.

For the x-axis solution each row of strips in an x = constant plane constitute a capacitive susceptance jB. Thus the equivalent circuit for modes propagating in the x direction is a transmission line loaded at intervals s by shunt susceptances jB. The propagation phase constant βn_x in the x direction is a solution of

$$\cos \beta n_x s = \cos hs - \frac{B}{2} \sin hs, \qquad (7)$$

where

$$h^2 = \left[k_0^2 - (\beta n_y)^2 \right] \cos^2 \theta_i'$$

and θ_i' is the angle of propagation, relative to the x-axis, for the TEM mode between strips when $n_y = 0$. Alternatively h may be expressed as $k_0 \cos\theta_x$ where $\cos\theta_x$ is the direction cosine between the x axis and the wave normal of the TEM wave existing between adjacent gratings, an expression valid for all n_y. In the analysis of the problem βn_y and βn_z are usually specified and hence

$$h^2 = k_0^2 - (\beta n_y)^2 - (\beta n_z)^2, \qquad (8)$$

since for the TEM mode, between gratings, the magni-

[8] J. Howes and E. A. N. Whitehead, "The Refractive Index of a Dielectric Loaded with Thin Metal Strips," Elliot Brothters Ltd., London, England, Elliot Bros. Rept. 119; July, 1949.
N. Kolettis, "Conformal Mapping Solution for Equivalent Relative Permittivity of an Artificial Strip Dielectric Medium," Case Inst. Tech., Cleveland, Ohio, Scientific Rept. No. 2, issued on Contract AF 19(604)-3887; January 15, 1959.

tude of the propagation phase constant must equal k_0. An approximate value of B is given by[9]

$$B = \frac{2k_0 d \cos\theta_i'}{\pi}\left\{ ln \csc\frac{\pi b}{2d} + \frac{1}{2}\,\frac{(1-\alpha^2)^2[(1-\alpha^2/4)(A_+ + A_-) + 4\alpha^2 A_+ A_-]}{(1-\alpha^2/4) + \alpha^2(1+\alpha^2/2-\alpha^4/8)(A_+ + A_-) + 2\alpha^6 A_+ A_-}\right\}, \quad (9)$$

where

$$\alpha = \sin \pi b/2d$$

and

$$A_{\pm} = \left[1 \pm \frac{k_0 d}{\pi}\sin\theta_i' - \left(\frac{k_0 d\cos\theta_i'}{2\pi}\right)^2\right]^{-1/2} - 1.$$

In this equation, $k_0 \cos\theta_i' = h$ and $k_0 \sin\theta_i' = (k_0^2 - h^2)^{1/2}$ and h is given by (8) in general. When $n_y = 0$, θ_i' is the angle of incidence relative to the x axis in Fig. 1, but for $n_y \neq 0$, k_0^2 is to be replaced by $k^2 = k_0^2 - (\beta n_y)^2$ and $h = k_0 \cos\theta_i'$ becomes $(k_0^2 - \beta^2 n_y^2)^{1/2} \cos\theta_i'$ while $k_0 \sin\theta_i'$ becomes $(k_0^2 - \beta^2 n_y^2)^{1/2} \sin\theta_i' = (k_0^2 - \beta^2 n_y^2 - h^2)^{1/2} = \beta n_z$. Once βn_x has been found from (7) the equivalent dielectric constant κ_e may be obtained from (6) since βn_z and βn_y are determined by the known incident wave on the medium.

Experimental Results

Measurement of the propagation constant βn_z was carried out for six different samples three of which were eight periods long, while the other three were ten periods long. The samples were constructed by mounting metallic foil strips between sheets of polyfoam as illustrated in Fig. 3. Each sample was made to fit inside a rectangular guide of internal dimensions 1 7/8″ by 7/8″ and having a removable top plate with a centered slot. The foil strips were bent over along the sides of each sample in order to make a reasonably good contact with the side walls of the guide. At each end of the sample the guide was closed off by a short-circuiting block so as to form a resonant cavity. The cavity was excited through a small circular aperture in one end plate and with an E_{11} mode incident in the main guide. When the cavity resonates $\beta n_z = n\pi/Nd$ where n is the nth resonant mode and N is the total number of sections in the sample. The available pass band of the main guide for the E_{11} mode permitted from 4 to 7 resonant frequencies to be found. Resonance was detected by means of a probe inserted into the cavity through the centered slot along the top surface of the cavity.

The measured values of βn_z for strips mounted on polyfoam may be converted to equivalent results for strips in free space by frequency scaling. If f_1 is a resonant frequency for the cavity filled with strips mounted on polyfoam, then $\kappa_p^{1/2} f_1$ is the resonant frequency of the same mode for a cavity filled with strips surrounded by free space. κ_p is the dielectric constant of polyfoam, about 1.08 in the experiments carried out. This frequency scaling was used to convert all results to that for strips surrounded by free space, so as to avoid the need to consider the properties of the supporting medium in the comparison of the theoretical and experimental results.

In the z-axis solution for the loaded guide $\beta n_x = \pi/d_1$ and $\beta n_y = \pi/d_2$ where d_1 is the guide height (7/8″) and d_2 is the guide width (1 7/8″). In the x-axis solution βn_x and βn_y are also fixed by the guide dimensions and hence (7) and (9) must be solved simultaneously for the parameter h. Eq. (8) is then readily solved for βn_z. The solution for h was obtained by plotting B vs h as determined by (7) and (9). The point of intersection of the two curves determines the value of h that satisfied both equations.

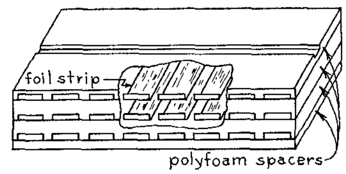

Fig. 3—Typical strip medium sample with 3 layers and 8 sections.

In Figs. 4 and 5 the theoretical and experimental values of βn_z are plotted as a function of frequency. The frequency has been scaled so that the results are for strips surrounded by free space. For Fig. 4 the parameters of the three samples measured are $a = 1.143$ cm, $b = 0.381$ cm, $s = 0.554$ cm, 0.739 cm, 1.11 cm, respectively and $N = 8$, *i.e.*, the samples are 8 sections long. The samples for Fig. 5 are 10 sections long and $a = 0.762$ cm, $b = 0.254$ cm, and $s = 0.554$ cm, 0.739 cm, and 1.11 cm, respectively. An examination of the experimental results shows very clearly that for small values of s the experimental values lie very close to the theoretical curve based on the z-axis solution, while for the largest value of s used the experimental results lie close to the theoretical curve based on the x-axis solution. Fig. 6 is a plot of the computed value of κ_e [based on (6)] as a function of frequency for the two samples for which $s = 0.739$ cm. At the higher frequencies κ_e rises rapidly in value since the edge of the first pass band is being approached.

[9] N. Marcuvitz, "Waveguide Handbook," McGraw-Hill Book Co., Inc., New York, N. Y., vol. 10 of M.I.T. Rad. Lab. Series, Sec. 5.18, Eq. (1a); 1951.

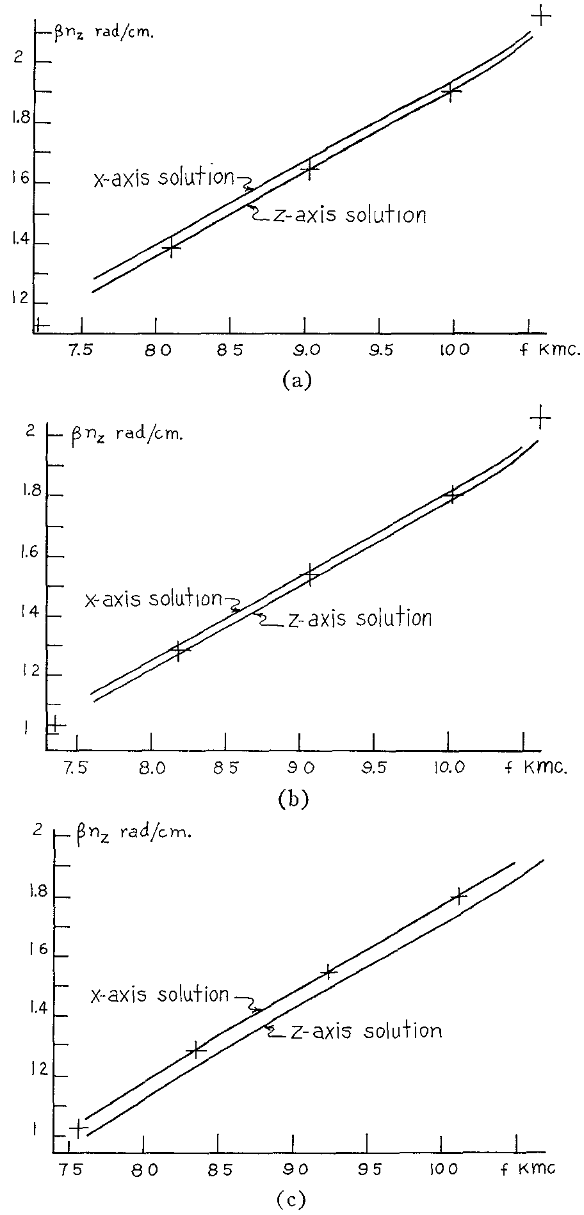

Fig. 4—Computed and measured values of βn_z for $a=1.143$ cm, $b=0.381$ cm, + experimental points. (a) $s=0.554$ cm, (b) $s=0.739$ cm, (c) $s=1.11$ cm.

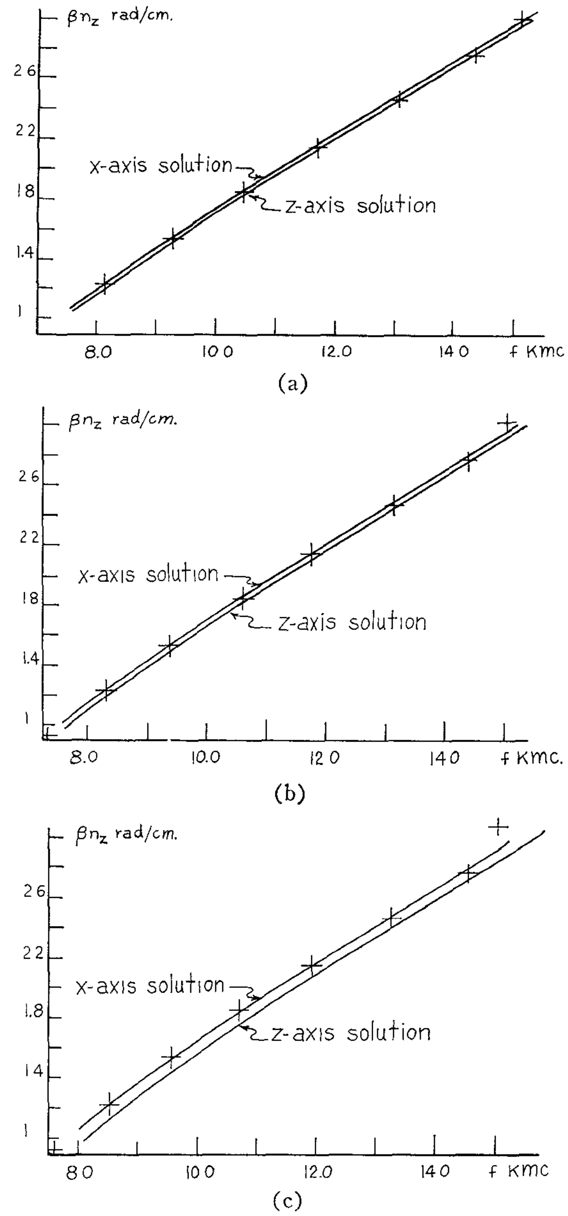

Fig. 5—Computed and measured values of βn_z for $a=0.762$ cm, $b=0.254$ cm, + experimental points. (a) $s=0.554$ cm, (b) $s=0.739$ cm, (c) $s=1.11$ cm.

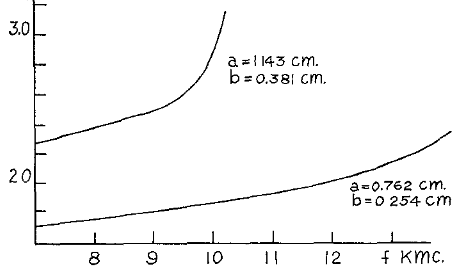

Fig. 6—Computed value of κ_e for two samples for which $s=0.739$ cm, based on experimental values of βn_z from Figs. 4(b) and 5(b).

The theoretical solution for βn_z, in the case $n_y=0$, yields βn_z as an implicit function of the free space wave number k_0. When $n_y \neq 0$ the solution for βn_z at any given frequency f_1 may be found by simply replacing $k_0=2\pi f_1(\mu_0\epsilon_0)^{1/2}$ by $k=(k_0^2-\beta^2 n_y^2)^{1/2}$. Thus the curves presented in Figs. 4, 5, and 6 as a function of frequency with $\beta n_y=\pi/d_2$, may be interpreted as results for the case $n_y=0$, *i.e.*, propagation in xz plane only, by replacing the frequency scale f by a new frequency scale f_1 where

$$f_1 = \left[f^2 - \frac{1}{4\mu_0\epsilon_0 d_2^2}\right]^{1/2}.$$

Conclusions

Experimental results have been presented which show that for small strip spacing s, the z-axis solution gives good results for the propagation phase constant of the strip-type artificial dielectric medium. For larger values of the spacing s, a solution based on modes propagating normal to the broad face of the strips gives good results. A combination of the two methods thus provides a method of estimating the value of the propagation constant without having to resort to the laborious procedure of taking higher-order mode interaction into account. In order to estimate the best value of the propagation phase constant it is necessary to plot βn_z as a function of the strip spacing s with other parameters fixed. When this is done it is found that for a certain range of s, the two methods give results that are in close agreement. For smaller values of s the best estimate curve of βn_z should be drawn so as to approach the z-axis solution, while for large values of s the best estimate curve for βn_z should be made to approach the x-axis solution. Typical results that are obtained by this procedure are given in Collin.[4]

From a knowledge of the propagation constant βn_z, an equivalent dielectric constant for the medium, along the z and y axis, may be defined and evaluated by considering an analogy between the strip-type artificial dielectric and a homogeneous uniaxial anisotropic dielectric medium. Some of the difficulties encountered in formulating such an analogy were pointed out. In particular, it was shown that a complete analogy with a homogeneous anistropic dielectric medium was not possible in general.

Appendix

The strip-medium could be assumed to have a dielectric constant of unity in both the x and y directions. In this case one would attempt to make an analogy with a uniaxial medium with dielectric constants 1, 1, κ_e along the x, y and z axis. The z-axis now becomes the optic axis. In this case the eigenvalue equation for the homogeneous dielectric medium is

$$\beta^2 = \frac{\kappa_e k_0^2}{n_z^2\kappa_e + (1 - n_z^2)}.$$

Only if $n_y=0$ so $n_z^2=1-n_x^2$ is this equation the same as (5). Since (5) arises naturally from the low-frequency limit of (1) it seems preferable to use the analogy discussed in the paper.

A further point of interest is that if the analogy discussed in this Appendix is used then in place of (6), the dielectric constant is in general given by

$$\kappa_e = \frac{(\beta n_x)^2 + (\beta n_y)^2}{k_0^2 - (\beta n_z)^2},$$

which is not the same as that given by (6). The analogy discussed in this Appendix is the same as that used by Kolettis.[10]

Although the two analogies lead to different results for the equivalent dielectric constant κ_e, they are both of equal usefulness in practice since, invariably, the parameter κ_e is used to compute β and, provided the appropriate eigenvalue equation is used, the same value of β obviously results. Clearly, since the strip medium is not fully analogous to a uniaxial homogeneous anisotropic dielectric medium, not too much physical significance can be attached to the defined equivalent dielectric constant in either case.

[10] N. J. Kolettis, "Electric Anisotropic Properties of the Metallic-Strip-Type Periodic Medium," Case Inst. Tech., Cleveland, Ohio, Scientific Rept. No. 17 issued on Contract No. AF 19-(604)-3887; October, 1960.